XLIII. *A Demonstration of Two Theorems mentioned in Art.* XXV. *of the* Philosophical Transactions *for the Year* 1775. *In a Letter from* Charles Hutton, *Esq.* F. R. S. *to the Rev. Dr.* Horsley, *Sec. R. S.*

TO THE REV. DR. HORSLEY, SEC. R. S.

REV. SIR,

Woolwich Warren, Royal Mil. Acad. April 17, 1776.

R. June 27, 1776.

THE following is a demonstration of the two theorems mentioned in Art. XXV. of the Philosophical Transactions for the year 1776. If you think them proper to be inserted in the next volume of Transactions, be pleased to communicate them to the Royal Society for that purpose, from, &c.
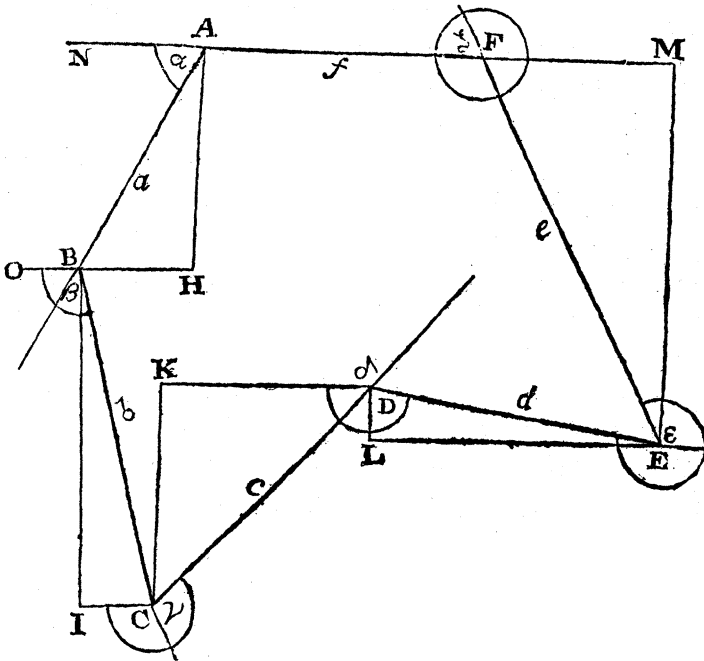
THE DEMONSTRATION.

Parallel to one ſide, AF, of the polygon ABCDEF, through all the angular points, draw BH, CI, DK, EL, FM. Then are the ſeveral angles, which are made by theſe parallels and the adjacent ſides of the figure, reſpectively equal to the ſums of all the exterior angles of the figure, from the beginning to the place of each angle; by the beginning is meant either extremity of the ſide AF. Thus the $\angle$ NAB $= \alpha$, the $\angle$ OBC $= \alpha + \beta$, the $\angle$ ICD $= \alpha + \beta + \gamma$, the $\angle$ KDE $= \alpha + \beta + \gamma - \delta$ (for here δ, being without the figure, is ſubtracted inſtead of being

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added), the $\angle$ LEF $= \alpha + \beta + \gamma - \delta + \varepsilon$, &c. All this is evident from Eucl: I. 29.; the angles we fpeak of being thofe, which are meafured by the little arcs defcribed about each angular point in the figure.

I. If right lines, AH, BI, CK, DL, EM, be drawn from the angular points, perpendicular each to the parallel which paffes through the next angular point, the fums of the perpendiculars, drawn in contrary directions, the one upwards, and the other downwards, will be equal. And each perpendicular will be a fourth in proportion with the radius, that fide of the polygon which is adjacent both to the perpendicular and the parallel on which it falls, and the fine of the fum of the external angles taken to that inclufively from which the perpendicular is drawn. Thus, AH $+$ BI $+$ DL $=$ CK $+$ EM; and rad. fin. $\alpha =$ AB : AH; and rad. fin. $\overline{\alpha + \beta} =$ B C : BI, and in like manner of the reft. Take the value, therefore, of each perpendicular by thefe analogies, putting unity for the radius, fubtract the fum of all that are drawn upwards from the fum of all that are drawn downwards, and the remainder, put equal to o, is the firft equation; that is, AH $= a \times$ f. α (for $\angle$ ABH $= \angle \alpha$); BI $= b \times$ f. $\overline{\alpha + \beta}$ (for f. $\angle$ ICB $=$ f. $\angle$ HBC $=$ f. of its fuppl. OBC or $\alpha + \beta$); in like manner, CK $= - c \times$ f. $\overline{\alpha + \beta + \gamma}$; DL $= d \times$ f. $\overline{\alpha + \beta + \gamma - \delta}$; EM $= - e \times$ f. $\overline{\alpha + \beta + \gamma - \delta + \varepsilon}$, &c. the laft perpendicular will always be $=$ o, becaufe the fine of 360°, or of $\alpha + \beta + \gamma - \delta + \varepsilon + \zeta$ is nothing. Hence $a \times$ f. $\alpha + b \times$ f. $\overline{\alpha + \beta} + c \times$ f. $\overline{\alpha + \beta + \gamma} + d \times$ f. $\overline{\alpha + \beta + \gamma - \delta} + e \times$ f. $\overline{\alpha + \beta + \gamma - \delta + \varepsilon} =$ AH $+$ BI $-$ CK $+$ DL $-$ EM $=$ o, which is the firft equation.   II.
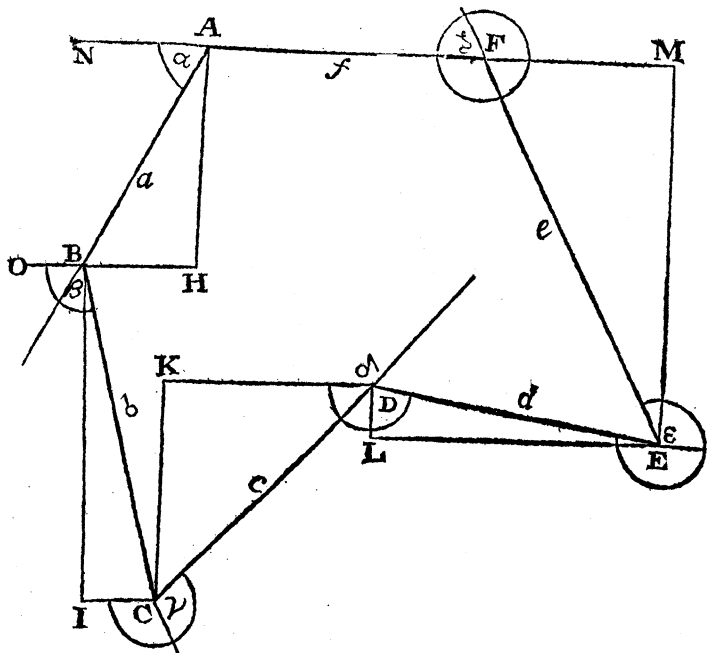
II. In like manner it appears, that the intercepted parts BH, CI, DK, EL, FM, AF, of the parallels before-mentioned, are equal to the feveral correfponding fides drawn into the cofines of the fame fums of the exterior angles (the radius being 1.); and becaufe $BH - CI - DK - EL + FM + AF = 0$, therefore $a \times \text{cof.}\,\alpha + b \times \text{cof.}\,\overline{\alpha + \beta} + c \times \text{cof.}\,\overline{\alpha + \beta + \gamma} + d \times \text{cof.}$

$\overline{\alpha + \beta + \gamma - \delta} + e \times \text{cof.}\,\overline{\alpha + \beta + \gamma - \delta + \varepsilon} + f \times \text{cof.}\,\overline{\alpha + \beta + \gamma - \delta + \varepsilon + \zeta}$

$= 0$, which is the fecond theorem. Or, for the laft term, $f \times \text{cof.}\,\overline{\alpha + \beta + \gamma - \delta + \varepsilon + \zeta}$, of this latter theorem, might be fubftituted its value f only.

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